variation of ρ and transport properties with temperature through the boundary layer.

The authors suggest, on the basis of experimental data, the following turbulent boundary-layer relations for air in the range $0.6 < T_w/T_{t\infty} < 1.3$:

$$Nu = Nu_i(T_w/T_{t\infty})^{-0.25}. \quad (8)$$

Outside this range, or for other gases, a numerical solution of the turbulent boundary-layer equations as described in this paper will give a good indication of the dependence of the Nusselt number on wall-to-gas temperature ratios.

Acknowledgements—The authors are grateful to Rolls-Royce Limited and the S.E.R.C. for their support.

REFERENCES

1. W. M. Kays and M. E. Crawford, *Convective Heat and Mass Transfer*. McGraw-Hill, New York (1980).
2. E. R. G. Eckert, Engineering relations for friction and heat transfer to surfaces in high velocity flows, *J. aeronaut. Sci.* **22,** 585–587 (1955).
3. L. W. B. Brown, Private Communication (1980).
4. L. Neal and M. H. Bertram, NASA TN-3969 (1967).
5. T. K. Bose, Some numerical results for compressible turbulent boundary-layer heat transfer at large free-stream/wall temperature ratios. In *Studies in Heat Transfer* (*A Festschrift for E. R. G. Eckert*), pp. 69–81. Hemisphere, Washington (1979).
6. P. J. Loftus and T. V. Jones, The effect of temperature ratios on the film cooling process, *J. Engng Pwr* **105,** 615–620 (1983).
7. T. Cebeci, Calculation of compressible turbulent boundary layers with heat and mass transfer, *A.I.A.A. Jl* **9,** 1091–1098 (1971).
8. T. Cebeci and A. M. O. Smith, *Analysis of Turbulent Boundary Layers*. Academic Press, New York (1974).
9. F. G. Blottner, Computational methods for inviscid and viscous two- and three-dimensional flow fields, AGARD LS-73 (1975).
10. L. Lees, Laminar heat transfer over blunt-nosed bodies at hypersonic flight speeds, *Jet Propulsion* **26,** 259–269 (1956).
11. *CRC Handbook of Tables for Applied Engineering Science*. CRC Press, Cleveland, OH (1976).
12. T. V. Jones, D. L. Schultz and A. D. Hendley, On the flow in an isentropic light piston tunnel, A.R.C. Reports & Memoranda No. 3731 (1973).
13. D. L. Schultz and T. V. Jones, Heat transfer measurements in short duration hypersonic facilities, AGARD AG-165 (1973).

Int. J. Heat Mass Transfer. Vol. 29, No. 1, pp. 164–167, 1986
Printed in Great Britain
0017-9310/86 $3.00 + 0.00
Pergamon Press Ltd.

Effect of suction on heat transfer rates from a rotating cone

K. Himasekhar and P. K. Sarma
College of Engineering, Department of Mechanical Engineering, Andhra University, Visakhapatnam—530 003 (A.P.), India

(*Received* 3 *August* 1983 *and in final form* 25 *June* 1985)

INTRODUCTION

Investigations on rotating systems have attained immense technological importance and, in that context, heat transfer systems from various types of axisymmetric bodies are presented in the literature [1]. Kreith [2] investigated the problem of cones and disks in turbulent and mixed flow. Sparrow and Gregg [3] analysed theoretically the problem of laminar heat transfer from a rotating disk with suction applied to the wall. Their analysis was restricted purely to the forced convective conditions and buoyancy effects were not considered. Hartnett and Deland [4] solved the problem of forced convection from rotating non-isothermal disks and cones with the intention of studying the Prandtl number effects on heat transfer rates. Herring and Grosh [5] studied heat transfer rates from a cone to air with the inclusion of buoyancy forces. Bergles [6, 7] classified rotation of the heat transfer surface as an active augmentation technique. The present investigation deals with a compound technique, namely rotation with simultaneous application of suction at the surface of a right vertical inverted cone, to assess the degree of augmentation achieved by treating the problem in its general form from which the special cases [1, 4, 5] can be arrived at.

FORMULATION OF THE PROBLEM

The configuration and disposition of the rotating cone with the coordinate system is the same as that given in ref. [5] save for application of suction of a constant value at the surface of the cone. The dimensionless boundary-layer equations for a steady, non-dissipative, constant property, axisymmetric flow are as follows:

Law of continuity:

$$2F + H' = 0. \quad (1)$$

Conservation of momentum:

x-direction (meridional)

$$F'' - (H - \beta_s)F' - F^2 + G^2 + \left(\frac{Gr}{Re^2}\right)\theta = 0 \quad (2)$$

y-direction (tangential)

$$G'' - (H - \beta_s)G' - 2FG = 0. \quad (3)$$

Energy equation:

$$\theta'' - Pr[(H - \beta_s)\theta' + F\theta] = 0. \quad (4)$$

The following velocity, temperature and dimensionless spatial functions are employed to arrive at the similarity transformations, i.e. equations (1)–(4):

$$u = x\omega \sin \alpha F(\eta) \quad (5)$$

$$v = x\omega \sin \alpha G(\eta) \quad (6)$$

$$w = (\nu\omega \sin \alpha)^{1/2}[H(\eta) - \beta_s] \quad (7)$$

$$(T - T_\infty) = (T_w - T_\infty)\theta(\eta) \quad (8)$$

$$\eta = (\omega \sin \alpha/\nu)^{1/2} z \quad (9)$$

$$Gr = g \cos \alpha (T_w - T_\infty)\beta x^3/\nu^2 \quad (10)$$

$$Re = \omega \cos \alpha x^2/\nu. \quad (11)$$

When $\beta_s = 0$, equations (1)–(4) would be identical to those solved by Herring and Grosh [5]. The above equations are to

NOMENCLATURE

C_{fy}	friction coefficient, $\tau_y/(\rho V^2/2)$
C_m	moment coefficient, $M/(\rho V_0^2 r_0^3/2)$
C_p	specific heat of the fluid
F, G, H	variables as defined in equations (5)–(7)
Gr	local Grashof number, $g\beta\cos\alpha(T_w-T_\infty)x^3/v^2$
g	gravitational acceleration
h	local heat transfer coefficient, $q/(T_w-T_\infty)$
k	thermal conductivity
L	cone slant height
M	torque for rotation
Nu	local Nusselt number, hx/k
Pr	Prandtl number, $\mu C_p/k$
q	heat flux
Re	local Reynolds number, Vx/v
r	cone radius
r_0	cone radius at the base
T	temperature of the fluid
T_∞	ambient temperature
T_w	surface temperature of the cone
V	local cone surface velocity, $x\omega\sin\alpha$
V_0	cone surface velocity at the base
u, v, w	velocity components along x, y, z, respectively
x, y, z	coordinate directions.

Greek symbols

α	cone apex half-angle
β	coefficient of thermal expansion
β_s	suction parameter
η	dimensionless coordinate, $(\omega\sin\alpha/v)^{1/2}z$
θ	dimensionless temperature, $(T-T_\infty)/(T_w-T_\infty)$
μ	dynamic viscosity
v	kinematic viscosity
ρ	density of the fluid
τ_y	circumferential shear stress
ω	angular veloxity.

Superscripts

$', '', '''$	first, second and third derivatives with respect to η.

be solved subject to the following boundary conditions:

$$\text{at}\quad z=0,\quad u=w=(T-T_w)=(v-x\cos\alpha\omega)=0 \tag{12a}$$

$$\text{as}\quad z\to\infty,\quad u=0,\quad v=0,\quad T=T_\infty. \tag{12b}$$

In terms of the variables defined by equations (5)–(9), these conditions become:

$$\eta=0,\quad F=H=0\quad\text{and}\quad G=\theta=1 \tag{13a}$$

$$\eta=\infty,\quad F=0,\quad G=0,\quad \theta=0. \tag{13b}$$

The local heat transfer rates are calculated from the Fourier equation:

$$q=-k\left.\frac{\partial T}{\partial z}\right|_{z=0}=h(T_w-T_\infty)$$

or in dimensionless form

$$\frac{Nu}{Re^{1/2}}=-\theta'(0). \tag{14}$$

The local friction factor $0.5\,C_{fy}=\tau_y/\rho(x\omega\cos\alpha)^2$ is given by

$$0.5C_{fy}\,Re^{1/2}=G'(0). \tag{15}$$

The dimensionless moment coefficient is obtained by

$$C_m=0.5M/\rho(L\omega\cos\alpha)^2r_0^3 \tag{16}$$

where M is the shaft torque required to overcome the shear of the rotating cone and is given by

$$M=-\int_0^L r\tau_y 2\pi r\,dx \tag{17}$$

and r_0 is the cone radius at the base where $x=L$. Equation (16) in terms of the non-dimensional variable becomes

$$C_m\,Re^{1/2}\sin\alpha/\pi=-G'(0). \tag{18}$$

NUMERICAL SCHEME

Equations (1)–(4) along with the boundary conditions (13a) and (13b) are solved by a finite-difference technique employing the Gauss–Siedel method [10]. The convergence criterion used for attaining the final converged solution is of the form $|G_i^{n+1}-G_i^n|_{max}<\varepsilon$ where the superscript refers to the number of iterations and the subscript refers to the grid location. The value of ε was varied to ensure a negligible dependence of the solution obtained of its value and was finally specified as 10^{-4}.

A comparison of the results obtained for $Pr=0.7$ with suction parameter equal to zero with those of Herring and Grosh [5], showed an excellent agreement confirming the validity of the scheme employed. Numerical computations were performed on an IBM-1130 computer and solutions are obtained for a wide range of controlling parameters. Some of the typical results obtained are discussed below.

RESULTS AND DISCUSSION

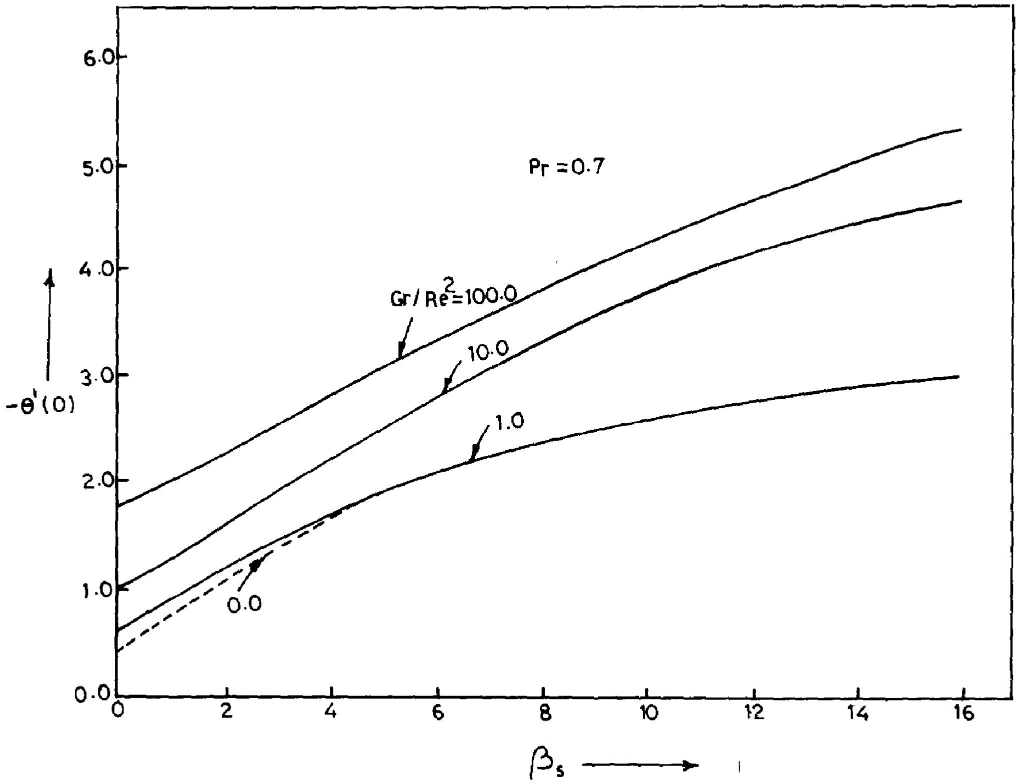
From the results it is observed that the suction parameter β_s has a profound influence on the tangential, circumferential and normal velocities.

As one would expect, the hydrodynamic boundary-layer thickness decreases with increase of the suction value at the permeable wall of the rotating cone. Furthermore, it is observed that the maximum tangential velocity gets reduced steeply with the increase in the value of β_s. Variation of circumferential velocity for $Gr\to 0$ in relation to β_s establishes that the effect of rotation is essentially confined to the very proximity of the wall for higher values of suction. The component of velocity normal to the rotating surface reveals that for values of $\beta_s>2$ the velocity field is independent of spatial coordinate and equals $-\beta_s$. The temperature field near the cone surface for different values of suction indicates that the thermal boundary-layer thickness is affected profoundly by the magnitude of the suction applied at the wall and as β_s increases, the thickness of the thermal boundary layer decreases resulting in steep thermal gradients. To conserve space the temperature and velocity fields are briefly described without depicting them in the figures.

The heat transfer results are shown in Figs. 1 and 2 for $Pr=0.7$ and 6.7, respectively, and for different flow situations, namely:

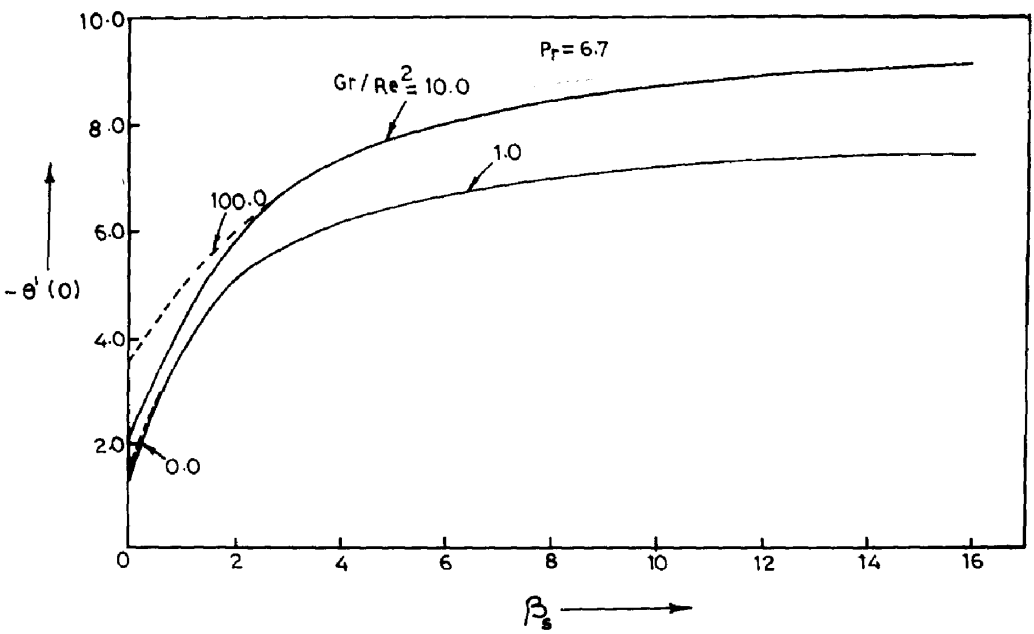
(i) pure forced convection, i.e. $Gr/Re^2=0.0$
(ii) mixed convection, i.e. $Gr/Re^2\simeq 1.0$
(iii) free convection, i.e. $Gr/Re^2\gg 1$.

From Fig. 1, it is evident that for $Pr=0.7$ and $Gr/Re^2=0$ and 1, $[-\theta'(0)]$ has identical variation with respect to β_s except for small values of β_s. However, for $Gr/Re^2>10$, it is observed that $[-\theta'(0)]$ increases with the increase in Gr/Re^2 for a given value of β_s. The heat transfer rates from the rotating cone to water (i.e. $Pr=6.7$) are shown in Fig. 2. In comparison with Fig. 1, the trends observed for this case are markedly different with regard to the influence of the parameter Gr/Re^2 on thermal gradient. For all values of $Gr/Re^2<1$, $[-\theta'(0)]$ is independent of Gr/Re^2 which implies that the influence of buoyancy forces on the velocity and temperature fields is negligible and the mechanism of heat transfer is dictated by



Pr = 0.7
Gr / Re² = 100.0
10.0
1.0
0.0
-θ'(0)
β_s
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
2
4
6
8
10
12
14
16

FIG. 1.



Pr = 6.7
Gr / Re² = 10.0
1.0
100.0
0.0
-θ'(0)
β_s
10.0
8.0
6.0
4.0
2.0
0
2
4
6
8
10
12
14
16

FIG. 2.

Table 1. Tabulation of $\theta'(0)$ for various values of β_s

β_s	$Pr = 0.7$, $Gr/Re^2 = 0.0$		$Pr = 6.7$, $Gr/Re^2 = 0.0$		$Pr = 0.7$, $Gr/Re^2 = 1.0$		$Pr = 6.7$, $Gr/Re^2 = 1.0$		$Pr = 0.7$, $Gr/Re^2 = 10.0$		$Pr = 6.7$, $Gr/Re^2 = 10.0$		$Pr = 0.7$, $Gr/Re^2 = 100.0$		$Pr = 6.7$, $Gr/Re^2 = 100.0$	
	$-\theta'(0)$	$-G'(0)$	$-\theta'(0)$	$-G'(0)$	$-\theta'(0)$	$-G'(0)$	$-\theta'(0)$	$-G'(0)$	$-\theta'(0)$	$-G'(0)$	$-\theta'(0)$	$-G'(0)$	$-\theta'(0)$	$-G'(0)$	$-\theta'(0)$	$-G'(0)$
0.0	0.4316	0.6114	1.2060	0.6085	0.6114	0.8427	1.4022	0.7055	1.0083	1.3990	2.1000	1.0500	1.7930	2.4185	3.5478	1.7848
	0.4285*	0.6159*			0.6120*	0.8507*			1.0173*	1.4037*			1.7946*	2.4738*		
0.125	0.4613	0.6491	1.5695	0.6536	0.6416	0.8810	1.7281	0.7372	1.0538	1.4443	2.3761	1.0601	1.8234	2.4842	3.7352	1.7949
0.25	0.4927	0.6880	1.9396	0.1706	0.6724	0.9194	2.0614	0.7730	1.0874	1.4882	2.6614	1.0792	1.8532	2.5213	3.9197	1.8061
0.5	0.5602	0.7704	2.6357	0.8067	0.7354	0.9968	2.7032	0.8580	1.1558	1.5765	3.2312	1.1273	1.9130	2.5950	4.2782	1.8323
1.0	0.7166	0.9556	3.7262	1.0575	0.8668	1.1534	3.7494	1.0831	1.2961	1.7548	4.2769	1.2796	2.0335	2.7420	4.9419	1.9056
2.0	1.0725	1.3684	5.0211	1.6820	1.1448	1.4691	5.0260	1.6730	1.5902	2.1162	5.7784	1.7924	2.2775	3.0319	6.0203	2.1641
4.0	1.6560	2.0050	6.1623	2.6994	1.6666	2.0233	6.1630	2.5330	2.2101	2.8368	7.2894	2.8914	2.7748	3.5978	7.3330	2.9866
8.0	2.3340	2.6670	6.9612	4.0098	2.3355	2.6692	6.9613	4.0050	3.3160	4.0258	8.4284	4.4483	3.7630	4.6545	8.4336	4.4652
16.0	2.9474	3.2100	7.4444	5.3332	2.9476	3.2002	7.4444	5.3332	4.6690	5.3360	9.1469	6.1540	5.3076	6.1821	9.1474	6.1565
32.0	3.3940	3.5550	7.7122	6.4012	3.3940	3.5556	7.7124	6.4012	5.8950	6.4003	9.5544	7.6191	6.9162	7.6218	9.5545	7.6194
64.0	3.6722	3.7647	7.8535	7.1111	3.6722	3.7647	7.8537	7.1112	6.7879	7.1112	9.7722	8.6487	8.1755	8.6489	9.7722	8.6487

* Values taken from ref. [4].

the pure forced convective conditions. Furthermore, for $Gr/Re^2 > 10$ and $\beta_s > 3$, $[\theta'(0)]$ is again an independent function of Gr/Re^2 and varies monotonically with the value of β_s.

Entries in Table 1 indicate the salient derivatives of the velocities and temperature at the wall for different values of the suction parameter. $G'(0)$ physically signifies a scale of the friction coefficient defined by equation (15). The results indicate that $G'(0)$ is essentially independent of Gr/Re^2 for $\beta_s > 6$. For values of $\beta_s < 6$, the friction factor is found to be dependent on the characteristic dimensionless ratio Gr/Re^2. In the case of air, for all values of β_s, the friction coefficient is found to be strongly dependent on Gr/Re^2 especially when the buoyant forces play a significant role in comparison with the inertial forces. However, for the case of pure rotation and mixed flow conditions, the parameter Gr/Re^2 does not have a marked influence on the friction coefficient.

The results of $[-G'(0)]$ vs β_s are of practical utility in estimating the power requirement to maintain rotation of the cone with simultaneous suction of constant value at the permeable surface.

Thus the present note establishes the following salient points:

(1) Figures 1 and 2 and Table 1 can be employed to evaluate the augmentation index defined as the ratio of the rate of heat transfer to the power required for given conditions of suction, i.e. β_s at the wall.

(2) The theoretical investigation undertaken encompasses a variety of physical problems which can be considered as special cases of the more general problem tackled. Thus, the special cases that would emanate are as follows according to the value of (Gr/Re^2):

(i) $Gr/Re^2 = 0$, forced convection with or without suction
(ii) $Gr/Re^2 \simeq 1$, mixed convection with or without suction
(iii) $Gr/Re^2 \gg 1$, free convection from the cone surface with or without suction.

Acknowledgements—The authors thank the University Grants Commission, New Delhi for the Financial Support.

REFERENCES

1. F. Kreith, Convective heat transfer in rotating systems. In *Advances in Heat Transfer* (Edited by T. F. Irvine and J. P. Harnett), Vol. 5, pp. 129–251 (1968).
2. F. Kreith, *Proc. Heat Transfer Fluid Mech. Inst*, pp. 29–36. Stanford University Press, Stanford, CA (1966).
3. E. M. Sparrow and J. L. Gregg, Mass transfer, flow and heat transfer about a rotating disk, *Trans. Am. Soc. mech. Engrs*, Series C, *J. Heat Transfer* **82**, 294–302 (1960).
4. J. P. Hartnett and E. C. Deland, The influence of Prandtl number on the heat transfer from rotating non-isothermal disks and cones, *Trans. Am. Soc. mech. Engrs*, Series C, *J. Heat Transfer* **83**, 95–96 (1961).
5. R. G. Herring and R. J. Grosh, Laminar combined convection from a rotating cone, *Trans. Am. Soc. mech. Engrs*, Series C, *J. Heat Transfer* **85**, 29–34 (1963).
6. A. E. Bergles, Augmentation of two-phase heat transfer. In *Two-Phase Flows and Heat Transfer* (Edited by S. Kakac and T. N. Veziroglu), Vol. 2, 817–841 (1976).
7. A. E. Bergles, R. L. Webb, G. H. Junkham and M. K. Jensen, Bibliography on augmentation of convective heat and mass transfer, Bibliographic Report, Iowa State University, Iowa (1980).
8. S. Ostrach and W. H. Braun, Natural convection inside a flat rotating container, NACA Tech. Note No. 4323 (1958).
9. E. M. Sparrow and H. S. Yu, Local non-similarity thermal boundary-layer solutions, *Trans. Am. Soc. mech. Engrs*, Series C, *J. Heat Transfer* **93**, 328–334 (1971).
10. B. Carnahan, H. A. Luther and J. O. Wilkes, *Applied Numerical Methods*. John Wiley, New York (1969).
11. P. J. Roache, *Computational Fluid Dynamics*. Hermosa, Albuqerque, NM (1972).